

THE SURPRISE ACCOUNT OF MODAL KNOWLEDGE ACQUISITION

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Abstract

This paper argues that we use emotions to acquire our intuitive modal knowledge. In particular, it explores the role of surprise in knowledge of epistemic and circumstantial modal statements. It begins with a discussion of standard modal semantics which elucidates the sorts of epistemic problems that creatures like us face in their attempt to know modal statements. It then distinguishes various sorts of surprise and elucidates their fittingness conditions. After that, it formulates some surprise tests and exemplifies their scope and limits in modal knowledge acquisition. To initiate a surprise test, one supposes or imagines some proposition. Roughly, if that proposition gives rise to a surprise response, one judges it impossible; otherwise, one judges it possible.

Keywords: Modal Epistemology, Emotions, Surprise

1 Introduction

Every normal adult human has abundant modal knowledge. We know epistemic possibilities like that we might not meet climate goals before it's too late. We know circumstantial possibilities like that Barack Obama could've been a plumber. Modal epistemologists have made considerable progress in the investigation of potential methods of modal knowledge acquisition (e.g., Gendler & Hawthorne 2002; Fischer & Leon 2017). However, modal epistemologists are yet to investigate the role of emotions in modal knowledge acquisition. Despite his sentimentalist account of moral cognition, even Hume didn't give sentiments a role in modal cognition (Millican 2020).¹

There have been relevant developments elsewhere, however. In *Decision, Order, and Time in Human Affairs*, the heterodox economist G. L. S. Shackle (1961) made an almost forgotten attempt to link his notion of "potential surprise" to modal judgement (cf. Levi 1967; Spohn 2006, 2012). Roughly, potential surprise is a reaction of disbelief to propositions that one supposes rather than learns. This paper explores the role of potential surprise in modal knowledge acquisition. It formulates some "surprise tests" and exemplifies their scope and limits in modal knowledge acquisition. To initiate a surprise test, one supposes or imagines some proposition. Roughly, if that proposition gives rise to a potential surprise response, one judges it impossible; otherwise, one judges it possible. My goal in the paper is to *assume* standard realist Kratzerian semantics for modal statements (Kratzer 1977), to describe in a precise way the epistemological problems that this semantics gives rise to, and to show that a surprise-based modal epistemology can solve those problems. This is of interest to those with an antecedent commitment to standard realist semantics for modal statements, but has wider interest insofar as it shows that some of the epistemological problems that arise in that framework are solvable.²

Although most modal epistemologists investigate acquisition of metaphysical modal knowledge, some have begun to widen their attention to more quotidian types of modal knowledge (Vetter 2016; Ichikawa 2016; Roca-Royes 2017; Strohmingner & Yli-Vakkuri 2019; Boardman & Schoonen 2023, 2025; Jones & Schoonen 2026). This paper widens the explanandum further and investigates our acquisition of epistemic modalities in addition to circumstantial modalities. If, as we discuss in a moment, the truth-conditions of circumstantial and epistemic modal statements are equivalent to statements about what is compatible with what (Kratzer 2012), then it is worth exploring the conjecture that

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²A reviewer notes that the surprise-based modal epistemology might also fit well with certain forms of non-realist semantics (e.g., Blackburn 1993; Yalcin 2011). I agree, but note that one can endorse an emotion-based modal epistemology without endorsing (e.g.) an expressivist modal semantics. The more challenging task is to show that the surprise-based modal epistemology can be integrated with a realist semantics. My paper attempts to address that more challenging task.

our capacities to determine what is compatible with what underlie our knowledge of circumstantial and epistemic modal statements. If that conjecture is borne out, then there is a sense in which circumstantial and epistemic modal statements are known in similar ways (cf. Vetter 2016, 786–791). Of course, to know circumstantial and epistemic modal statements in similar ways isn’t to know them in the exact same ways. As Williamson (2016b, 799) puts it, “the distinctions are subtle, but can be made”.³

Here is the plan. §2 uses modal semantics to introduce the main epistemological problems that modal knowledge acquisition presents. §§3–4 discuss the formidable nature of those problems for creatures like us and our solutions to them. After that, §5 introduces some different types of surprise. §6 puts those types of surprise to work in some surprise tests that implement our solutions to the problems of modal knowledge acquisition.

2 Modal Semantics

Modal knowledge is knowledge of the propositions that modal statements express in context. Modal semantics informs us which propositions modal statements express in context. Modal semantics therefore sets the target that our methods of modal knowledge acquisition have to hit (Williamson 2020, 5). Given the target, modal epistemologists can start to appreciate the epistemological challenges that creatures like us face in their attempt to hit it.⁴ This section therefore assumes standard realist modal semantics—i.e., Kratzer’s (1977) premise semantics—and uses it to factorise the problem of modal knowledge acquisition into its component parts (cf. Ichikawa 2016; Phillips & Knobe 2018).

The short version of premise semantics is that modal statements have the form $\text{MODAL}(B)(p)$ (von Fintel & Gillies 2011, 78). MODAL is some quantifier Q (e.g., \exists or \forall) whose domain is the set of possible worlds. B is a “premise set” of propositions relevant in context. There are lots of different sorts of premise sets—e.g., some represent the laws, some represent the circumstances, some represent what is known, some represent our priorities (Portner 2009), and some represent a mix of these considerations (Knobe & Szabó 2013). The relevant premise set is determined by the speaker’s conversational context (Kratzer 1977). The premise set, B , determines an “accessible” subset of the set of possible worlds. This subset contains a possible world w just in case every proposition in B is true at w . Put otherwise, $\bigcap B$ is the set of accessible worlds.⁵ The prejacent proposition p is just that proposition whose modal status is at issue. $\text{MODAL}(B)(p)$ is true if and only if p is true at Q of the possible worlds in $\bigcap B$.

Here are some examples to illustrate. (1) is a circumstantial modal statement. It concerns what is possible in view of the relevant circumstances.

- (1) Hydrangeas can grow in Korea. (Kratzer 1977)

³Although this paper does not include any explicit discussion of our knowledge of metaphysical modalities, one can tease out the implications if one thinks that metaphysical modalities are the broadest circumstantial modalities (cf. Williamson 2016a, 454–456; Ichikawa 2016). I am neutral about these matters here.

⁴These observations relate to the requirement that epistemological theories integrate with semantic theories. For discussion, see Stalnaker (1968); Peacocke (1999); Williamson (2020); Roca-Royes (2021); Cariani (2021, xx–xxii).

⁵As an idealisation, suppose that propositions are sets of possible worlds. $\bigcap B$ intersects the propositions in B . It follows that $\bigcap B$ contains just those possible worlds in which every member of the premise set, B , is true.

In particular, the premise set of (1) consists of propositions about the relevant botanical and meteorological circumstances. The accessible possible worlds are those in which the relevant botanical and meteorological circumstances are true. An utterance of (1) says that at some possible world at which the relevant botanical and meteorological circumstances are true, the prejacent proposition *that at some future time, hydrangeas grow in Korea* is true.⁶

Further, (2) is an epistemic modal statement. It concerns what is possible in view of the relevant information.

(2) John might've done it.

For example, we can suppose that the premise set of (2) consists of the detective's beliefs in a murder case. The accessible possible worlds are those in which the detective's beliefs are true. An utterance of (2) by the detective says that at some possible world at which the detective's beliefs are true, the prejacent proposition *that John did it* is true.

Premise semantics helps modal epistemologists to appreciate the problems that modal knowledge acquisition presents. To know a modal statement one must have epistemic access to whether its truth-conditions are met. The truth-conditions of modal statements involve premise sets and points of evaluation (e.g., possible worlds) that are “displaced”—i.e., aren't here, now, or actual (von Fintel & Heim 2011). This suggests that knowers face two central cognitive problems. First, the problem of the premise set requires knowers to establish epistemic access to the premise set. Second, the problem of displacement requires knowers to establish epistemic access to the truth-values of propositions at accessible possible worlds that aren't actual. §§3–4 provide further discussion of these problems and outline hypotheses about our solutions to them.

3 The Problem of the Premise Set

Epistemic access to the premise set might at first seem to require knowledge of the premise set.

Knowledge of the Premise Set

An agent x has knowledge of the premise set B of a modal statement M iff for every proposition q that is a member of B , x knows that q is a member of B .

For example, an agent x has partial knowledge of the premise set of (1) if x knows that the proposition that hydrangeas are plants is part of the premise set of (1). However, given that the concept PREMISE SET is a technical concept from formal semantics, non-experts don't have that concept. Non-experts therefore can't even entertain propositions about premise sets.

An alternative hypothesis is that non-experts establish epistemic access to premise sets by accepting them.

Acceptance of the Premise Set

An agent x accepts the premise set B of a modal statement M iff x accepts the conjunction of B 's members $\bigcap B$.

⁶I am going to assume that the prejacent of a present tense circumstantial modal statement concerns the future (Rullmann & Matthewson 2018, 301). For in-depth discussion of the interaction between modal and temporal issues which supports that assumption, refer to Rullmann & Matthewson (2018).

The relevant notion of acceptance here is Stalnaker’s (1984, 79):

[a]cceptance [...] is a generic propositional attitude concept with such notions as presupposing, presuming, postulating, positing, assuming and supposing as well as believing falling under it. [...] To accept a proposition is to treat it as a true proposition in one way or another—to ignore [...] the possibility that it is false.

§6 discusses the specific attitudes we adopt to premise sets. The point for present purposes is that if one accepts a premise set B , then one has an attitude with the content $\bigcap B$ rather than an attitude *about* premise sets. Acceptance of a premise set therefore doesn’t require the possession of PREMISE SET.

However, non-experts often aren’t in a position to accept premise sets. The premise sets of circumstantial modal statements often include facts about the laws of nature which most non-experts cannot entertain. For example, the premise set of (1) includes the sorts of chemical and biological laws that botanists use to explain the life of plants. Further, these premise sets often include obscure scientific facts about the circumstances. For example, the premise set of (1) includes the obscure botanical fact that *Hydrangea serrata* contains phyllodulcin. Most non-experts aren’t in a position to entertain propositions of that sort. Given that we aren’t always in a position to accept what is known, similar points extend to our knowledge of the premise sets of epistemic modal statements.⁷

Acceptance of premise sets isn’t within our cognitive reach. But despite their failure to entertain the proposition that *Hydrangea serrata* contains phyllodulcin, lots of people know that hydrangeas can grow in Korea. More generally, non-experts have abundant knowledge of modal statements despite the fact that they aren’t in a position to accept their premise sets. It is therefore plausible that non-experts use a more heuristic approach to establish epistemic access to the premise set of a modal statement (Kahneman et al. 1982; Gigerenzer et al. 2011). A natural proposal is that to evaluate a modal statement, non-experts accept some set of propositions B^* that *approximates* its premise set B .

Approximation

A set of propositions B^* approximates B relative to a modal statement of the form $\text{MODAL}(B)(p)$ *only if* p is possible in view of B^* *iff* p is possible in view of B .⁸

The basic thought is that B^* approximates B relative to a modal statement M only if B^* and B have the same modal consequences with respect to M ’s prejacent p . The relativisation to a modal statement M captures that B^* needn’t have the same modal consequences as B with respect to every proposition whatsoever. Rather, what matters

⁷Since the extension of ‘what is known’ isn’t always one’s own knowledge (e.g., it is the group’s evidence in a murder investigation), denial of positive introspection (i.e., if you know, then you know that you know) isn’t crucial to that point. For the case against positive introspection, see Williamson (2000).

⁸Although it is possible to define a notion of approximation more suitable for knowledge of necessities, the present discussion restricts itself to possibilities and impossibilities. Note further that Approximation doesn’t give sufficient conditions. Let BIDEN = the singleton set that contains the proposition that Biden loves ice cream. The proposition that hydrangeas grow in Korea is possible in view of BIDEN , but intuitively BIDEN doesn’t approximate the premise set of (1). In contrast, knowledge that hydrangeas can grow in Korea in view of the fact that Biden loves ice cream doesn’t put one in a position to know (1). To give sufficient conditions we might therefore require in addition that knowledge of $\text{MODAL}(B^*)(p)$ puts one in a position to know $\text{MODAL}(B)(p)$.

is match of modal consequences with respect to the prejacent of the modal statement at issue. This proposal simplifies the problem of the premise set because non-experts often *are* in a position to accept some set of propositions B^* that approximates the premise set (cf. Williamson 2007, 147–148). For example, although we aren’t in a position to accept the laws of nature, we often are in a position to accept folk physics (Spelke 1990; Carey 2009).

This section began with the problem of the premise set: those that know modal statements must establish epistemic access to their premise sets somehow. Knowledge of premise sets is out of the question. Even acceptance of premise sets is difficult if not impossible for creatures like us to achieve. It is therefore plausible that we have some more heuristic solution to the problem of the premise set. My proposal is that creatures like us establish epistemic access to the premise set by accepting $\bigcap B^*$. $\bigcap B^*$ is the conjunction of a set of propositions B^* that approximates the premise set B .

4 The Problem of Displacement

To know a modal statement M , we must often establish epistemic access to facts about the truth-values of its prejacent in accessible possible worlds that aren’t actual (‘displaced worlds’, hereafter). This is either because we often have no epistemic access to the truth-value of M at the actual world or because the truth-value of the prejacent at the actual world doesn’t settle the truth of M itself. The fact that p has some truth-value at the actual world doesn’t establish that p has that truth-value at every accessible world (except if the actual world is the only accessible world). Further, if the actual world isn’t accessible (e.g., if the premise set consists of beliefs, one of which is false), then the actual truth-value of the prejacent isn’t relevant to the truth-conditions of M .⁹

This section argues that we can use cognitive capacities that detect *what is compatible with what* to establish epistemic access to the truth-values of propositions in displaced worlds. As I am going to explain, statements about what is compatible with what are equivalent to modal statements (Kratzer 1977). The appeal to equivalences might appear controversial because it doesn’t follow from the fact that we *can* use some cognitive capacities to acquire knowledge that we *in fact* use those cognitive capacities to acquire knowledge (Jenkins 2008). But these equivalences provide good abductive evidence that we in fact use one of the relevant capacities to acquire modal knowledge *if it is otherwise plausible that we use that capacity to acquire modal knowledge* (Williamson 2007, 2016b, 2021). I am therefore going to argue that an important additional feature of modal knowledge acquisition is that it is often *intuitive*. If we can pinpoint a cognitive capacity that can detect *what is compatible with what* and can capture the *intuitive* nature of modal knowledge acquisition, then there is reason to think that capacity can underlie modal knowledge acquisition. I am going to suggest that the cognitive capacity that underlies surprise has these features.

We begin with the equivalences. A core part of Kratzer’s (1977) premise semantics is the observation that a modal statement of the form MODAL (B) (p) is equivalent to a

⁹The problem can seem insurmountable if we assume that epistemic access to displaced worlds requires causal connection to them (e.g., McGinn 1975). However, few epistemologists now accept that knowledge requires causal connection to what is known (Lewis 1984). More generally, my strategy in this section is to attempt to clarify our epistemic access to accessible possible worlds that aren’t actual without relying on substantive assumptions about the metaphysics of possible worlds. As I am about to explain, this style of argument is due to Williamson (2007, 2021).

statement about whether its premise set B is *compatible with* its prejacent p .

Kratzer's Equivalences

- (i) A possibility statement of the form $\diamond(B)(p)$ is equivalent to the compatibility statement that its prejacent p is compatible with its premise set B .¹⁰
- (ii) An impossibility statement of the form $\neg\diamond(B)(p)$ is equivalent to the incompatibility statement that its prejacent p is incompatible with its premise set B .

These equivalences show that if some cognitive capacities can detect whether p is compatible with a set of propositions B regardless of the actual truth-value of p , then these capacities can detect whether p is possible in view of B regardless of the actual truth-value of p . Put otherwise, these equivalences show that cognitive capacities that detect what is compatible with what are sensitive to the truth-values of propositions in displaced worlds. Given these equivalences, we *can* use cognitive capacities that detect what is compatible with what to establish epistemic access to the truth-values of propositions in displaced worlds. But whether we in fact use these capacities to acquire modal knowledge is a function of whether these capacities can capture additional features of modal knowledge acquisition.

An important additional feature of modal knowledge acquisition is that it is often intuitive. The moral psychologist Jonathan Haidt (2001, 818) gives some instructive contrasts between intuitive and deliberative judgement. Intuitive judgements appear “suddenly and effortlessly in consciousness, without any awareness by the person of the mental processes that led to the outcome”. Intuitive judgements are such that “the outcome but not the process is accessible to consciousness”. In contrast, deliberative judgement “occurs more slowly, requires some effort, and involves at least some steps that are accessible to consciousness”. Many modal judgements fit Haidt’s stereotype of intuitive rather than deliberative judgement. If I consider whether Putin could’ve refrained from invading Ukraine, the answer comes immediately and effortlessly to me: he could’ve. Similarly, if I consider whether an infant could climb to the top of the Eiffel Tower, my judgement is immediate and effortless: it couldn’t. In forming these judgements, I am not aware of the mental processes that led to them. Although I can come up with reasons *post-hoc*, I wasn’t aware of considering any reasons in my initial judgement. Presumably I am not special in this respect. It is therefore reasonable to assume that we often have intuitive modal knowledge.¹¹

This presents modal epistemologists that are sympathetic to realist Kratzerian semantics with a difficult challenge. The cognitive capacities that underlie intuitive modal knowledge acquisition must detect what is compatible with what *and* present that information to the subject in the form of an intuition about the modal status of the relevant prejacent proposition. This is a challenge because it might seem that determination

¹⁰Kratzer (2012, 10) gives a set-theoretic definition of compatibility in terms of non-empty intersection (i.e., p is compatible with B iff $\bigcap(\{p\} \cup B) \neq \emptyset$). But an intuitive grasp of compatibility suffices in what follows.

¹¹As a reviewer observes, we might get evidence that intuitive modal judgements don’t amount to knowledge and that, to the extent we have modal knowledge, it is mostly due to our deliberative capacities. However, as modal epistemologists, our default position should be non-sceptical. An account that can show that intuitive modal judgements constitute knowledge is *ceteris paribus* preferable to an account that cannot show that. This is, of course, a methodological stance that is ultimately judged by its theoretical fruits.

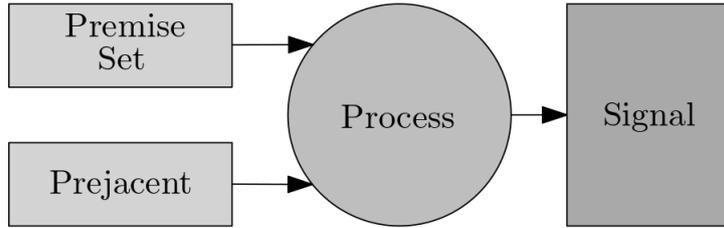


Figure 1: The functional architecture of the cognitive structure that implements our solution to the problem of displacement.

of what is compatible with what is the preserve of distinctively deliberative capacities. However, this paper argues that we can address this challenge if we recognise the role of emotions. My proposal is that creatures like us have an unconscious cognitive capacity which outputs an emotional signal that carries information about what is compatible with what and which presents itself to consciousness in the form of an intuition about the modal status of the relevant proposition (see Figure 1). This proposal might seem implausible if one assumes that the cognitive capacity that it posits is some exceptionalist module that functions to detect bizarre modal truths with no connection to the rest of our cognitive lives. But what if we can explain modal knowledge acquisition by appeal to a cognitive capacity that everyone recognises is central to human cognition? In §§5–6, I am going to argue that the surprise process in fact has these features. The surprise process functions to detect whether new information is compatible with prior expectations and presents this information to consciousness in the form of an emotional signal (i.e., the surprise signal). If we can run the surprise process “offline” by varying its inputs, we can use it to detect whether the prejacent p of a modal statement is compatible with an approximation B^* of its premise set B . The rest of the paper elaborates this proposal in detail. After discussion of some different types of surprise in §5, §6 outlines some surprise tests and exemplifies their role in modal knowledge acquisition.

5 Types of Surprise

This section introduces some different types of surprise. §5.1 begins with a discussion of surprise, expectation, violation of expectation, and correctness or “fittingness” conditions. §5.2 and §5.3 then discuss some further types of surprise and their fittingness conditions.

5.1 Surprise

Psychologists have often given surprise an important role in our mental lives (e.g., Darwin 1872; Ekman 1992). More recently, there has been an explosion of interest in surprise within cognitive science (e.g., Reisenzein et al. 2019; Munnich et al. 2019). The dominant approach is CEM—i.e., the cognitive evolutionary model of surprise (Reisenzein et al. 2019; Macedo & Cardoso 2019; Munnich & Ranney 2019). CEM has at its heart the view that surprise is an emotional reaction to violation of expectation, which serves the fundamental life task of expectation revision.¹² It hypothesizes that the function of the surprise process is to output a surprise signal just in case new information violates prior expectations. The function of the surprise signal itself is to trigger adaptive responses to

¹²The research programme goes back several decades. For example, see Reisenzein (2000); Teigen & Keren (2003); Pezzo (2003); Macedo et al. (2004).

violation of expectation. Expectation revision is the most important adaptive response to violation of expectation that the surprise signal motivates (Reisenzein et al. 2019, 52).

To illustrate, imagine that you expect no tigers at the river. But you now spot a tiger there. Fear might save you this time because it causes you to run. But surprise saves you thereafter. The new information that there is a tiger at the river violates your prior expectation of no tigers there. Your surprise process detects that and therefore outputs a surprise signal that causes you to revise your prior expectation of no tigers at the river. Because of that, you don't go to drink at that river anymore and that saves your life. In short, surprise is the starter pistol of expectation revision.

The talk of “processes” and “signals” in CEM raises questions about the underlying framework of emotions. I am going to assume a recent cognitive account of emotions known as ‘BDTE’—i.e., the belief-desire theory of emotions (Reisenzein 2009a,b; Miceli & Castelfranchi 2015). BDTE hypothesizes that emotions are the outputs of two innate mechanisms. The belief-belief comparator compares new information to prior expectations. Its function is to detect whether new information violates prior expectations. The belief-desire comparator compares new information to prior desires. Its function is to detect whether new information frustrates prior desires. This belief-desire comparator determines whether a surprise is pleasant, unpleasant, or neutral. Since the valence of surprise plays no role throughout the rest of this paper, we limit our attention to the value neutral signal of violation of expectation that the belief-belief comparator outputs. This valence neutral signal of violation of expectation is also the referent of the term ‘surprise’ in CEM (Reisenzein et al. 2019, 59–60).

5.1.1 Expectation

Expectation is a propositional attitude similar to belief (Lewis 1979, 513–514).¹³ The most distinctive feature of expectation is its intermediate strength (Benton & Turri 2014, 1862). A state has intermediate strength just in case it is weaker than outright belief and stronger than 0.5 credence.¹⁴ Expectation is weaker than outright belief because it is permissible to expect some propositions that one shouldn't outright believe. For instance, it is permissible to expect to lose fair large lotteries. Expectation is stronger than 0.5 credence because it is impermissible to expect that p and have a credence $n \leq 0.5$ in p .¹⁵ For instance, you shouldn't have 0.5 credence that a coin is going to land heads and expect it to land heads. Note that if one outright believes that p , then one ought to expect that p . But one can expect that p despite not outright believing that p . The phrase ‘mere expectation’ hereafter indicates that x expects that p but doesn't outright believe that p .¹⁶

¹³The relevant notion of expectation isn't “normative” (cf. Basu 2023)—e.g., you can normatively expect a kleptomaniac not to steal from you even if you expect them (in my sense of ‘expect’) to steal from you. Normative expectations are some sort of pro-attitude. In contrast, expectation proper has a mind-to-world direction of fit (e.g., Searle 1983). Note further that we can adopt attitudes of expectation to propositions about the past, present, and future (cf. Judge 2018). We can expect that dinosaurs were often large, that there are supermassive black holes, and that things are going to get worse.

¹⁴For my preferred account of outright belief, see Schulz (2021a,b). Schulz (2021b) argues that outright belief is the strongest belief state implied by knowledge. An intuitive grasp of outright belief suffices in what follows.

¹⁵Note that the context disambiguates between ‘ p ’ as a propositional variable and ‘ p ’ as a sentential variable.

¹⁶I am going to assume in what follows that our expectations are non-modal. This is just to demonstrate that even agents without prior modal expectations can use my surprise tests to acquire modal knowledge.

5.1.2 Violation of Expectation

The surprise signal indicates violation of expectation. But what is violation of expectation? We might start with a narrow sense of ‘violation of expectation’. A proposition p narrowly violates x ’s expectations *iff* x expects its negation $\neg p$.¹⁷ However, suppose that John expects his colleague to arrive at 9:15am. The narrow account predicts that the only proposition that violates John’s expectations is that that his colleague doesn’t arrive at 9:15am. But intuitively that isn’t the only proposition that violates John’s expectations. For example, the proposition that his colleague arrives at 7:50am violates John’s expectations. Similarly, the proposition that his colleague doesn’t arrive violates John’s expectations. Unless we assume that expectations are closed under entailment, the narrow sense of ‘violation of expectation’ won’t capture these cases. Hence, it is worthwhile to explore another option.

A more natural characterisation of violation of expectation has it that whether a proposition violates one’s expectations is a matter of whether that proposition is incompatible with one’s expectations.

Violation of Expectation

p violates x ’s expectations *iff* p is incompatible with x ’s expectations.¹⁸

This broader characterisation of violation of expectation captures the relevant cases. The proposition that John’s colleague arrives at 7:50am is incompatible with the proposition his colleague arrives at 9:15am. Similarly, the proposition that John’s colleague doesn’t arrive is incompatible with the proposition his colleague arrives at 9:15am. Notice further that since $\neg p$ is incompatible with p , a proposition p violates an agent x ’s expectations in the narrow sense only if p violates x ’s expectations in the broader sense. We therefore capture that the proposition that John’s colleague doesn’t arrive at 9:15am violates John’s expectations.^{19,20}

5.1.3 The Fittingness of Surprise

Emotions represent/construe/appraise their objects in certain ways. For example, fear at a person represents that person as dangerous. An emotion fits just in case its appraisal is correct. This is the alethic view of fittingness (D’Arms & Jacobson 2000; Tappolet 2011;

¹⁷Note that just because an agent x hasn’t previously considered whether p , it doesn’t follow that x doesn’t have a credence in $\neg p$ greater than 0.5. In other words, x can expect $\neg p$ even if x never considered whether p (Yalcin 2018, 25). For example, until now, I’ve never considered whether there are more than 1000 socks in my drawer, but I expect that there are not more than 1000 socks in my drawer. It is therefore consistent with the narrow view that an agent x that has never previously considered whether p can undergo surprise at p (i.e., if x has a credence in $\neg p$ greater than 0.5).

¹⁸To adapt this definition to offline expectation, we can just prepose ‘offline’ to ‘expectations’. Similarly, p violates x ’s outright beliefs *iff* p is incompatible with x ’s outright beliefs. This becomes relevant in §5.3.

¹⁹Some endorse the view that a proposition violates x ’s expectations just in case x didn’t expect that proposition (e.g., Kind 2021, 13133). But that view faces serious objections. For example, John might expect his colleague to arrive with pants, but has no expectation about the colour of his pants. It is implausible that the proposition that John’s colleague wears black pants violates John’s expectations.

²⁰This characterisation of violation of expectation captures its use in the scientific literature. Proponents of CEM often characterise unexpected events as “inconsistent with” background expectations (e.g., Reizenzein et al. 2012, 564). More importantly, the propositions that are said to violate a subject’s expectations are seldom the negations of propositions the subject is said to expect (e.g., Reizenzein et al. 2012, 566).

Rosen 2015). For example, Jane’s fear of Tarzan fits just in case he is dangerous. I am going to assume the alethic view in what follows.²¹

The alethic view gives us a route to the fittingness conditions of surprise. We start with a hypothesis about the appraisal that surprise makes. The alethic view then allows us to infer the fittingness conditions of surprise without further ado. CEM hypothesizes that surprise carries information about mental representations (Reisenzein 2009b, 15). In particular, x ’s surprise that p appraises that p violates x ’s expectations (Reisenzein et al. 2019, 54). Given that an emotion fits just in case its appraisal is correct, it follows that

Fitting Qualitative Surprise

x ’s surprise that p fits just in case p violates x ’s expectations.

Imagine that Jack expects that Jill will help him fetch a pail of water. But he later learns that Jill went to fetch a pail of beer instead. He therefore undergoes surprise that she won’t help him and his surprise fits because that proposition violates his expectations.

Like most emotions, surprise comes in degrees. The intensity of the surprise signal indicates *to what extent* new information violates prior expectations. CEM hypothesizes that x ’s degree d of surprise that p appraises that p violates x ’s expectations to a proportional degree d' (Reisenzein et al. 2019, 54). It follows that

Fitting Quantitative Surprise

x ’s degree d of surprise that p fits just in case d is proportional to the degree d' to which p violates x ’s expectations.

Imagine that Jack expects but doesn’t outright believe that Jill will return home to help him. If Jack undergoes a significant degree of surprise that Jill won’t return home, then his degree of surprise doesn’t fit. His degree of surprise constitutes an over-reaction. Given that there was a moderate violation of expectation, Jack should’ve had a moderate degree of surprise.²²

5.2 Potential Surprise

Some economists and formal epistemologists recognise the existence and importance of *potential surprise* (Shackle 1949; Levi 2004; Spohn 2012; Williamson 2020, 30). Whereas surprise is a reaction to propositions that one learns, potential surprise is a reaction to propositions that one supposes or imagines.²³ For present purposes, to suppose a proposition p is to “reuse” the BDTE belief-belief comparator “offline” by inputting into it a proposition that one didn’t learn.²⁴ If the BDTE belief-belief comparator detects that

²¹There are innumerable disagreements about the nature of fittingness (e.g., Howard 2018). But those disputes aren’t relevant to the arguments in this paper. My goal is just to distinguish between token emotions with correct appraisals and token emotions with incorrect appraisals. Doubters of the alethic view are free to reinterpret my fittingness conditions as “correctness conditions”.

²²§6.1 discusses the theoretical importance of significant surprise in more detail.

²³Potential surprise isn’t the surprise you would have or the surprise you imagine you would have, if you were to learn a proposition. Despite its name, “potential” surprise is a mental state that we in fact exhibit. Compare “conditional” belief. Several philosophers agree that conditional beliefs are states that agents exhibit (e.g., Stalnaker 1984; Joyce 1999; Bradley 2017).

²⁴Put otherwise, to suppose a proposition is to simulate learning it. In contrast, some more expansive notions of supposition involve simulation of subsequent belief revisions made to accommodate that proposition (e.g., Eva et al. 2022). We must therefore distinguish “offline learning” from “offline updating” and note that it is the former notion that is relevant here (Williamson 2020, 26).

p violates one’s expectations, then it outputs a potential surprise signal. For example, if you expect that Quine wasn’t secretly a gravedigger, and you suppose that Quine was secretly a gravedigger, then you are apt to react with potential surprise (which you might experience as an intuition of impossibility). Whether or not Quine was secretly a gravedigger, you can react with potential surprise if that supposition clashes with your expectations. Given that BDTE predicts the existence of emotional reactions to non-actual states of affairs that we imagine (Reisenzein 2009b, 8), potential surprise is consonant with the science of surprise.

To grasp the fittingness conditions of potential surprise, we have to think about the appraisal it makes. This isn’t difficult because potential surprise is just like surprise except that it’s a reaction to propositions one supposes rather than learns. Potential surprise therefore involves the same appraisal as surprise itself. By familiar reasoning, it therefore follows that

Fitting Qualitative Potential Surprise

x ’s potential surprise that p fits just in case p violates x ’s expectations.

Imagine that Quine was a gravedigger. For those of us that expect otherwise, potential surprise that Quine was a gravedigger fits.

Potential surprise comes in degrees. The intensity of potential surprise indicates *to what extent* one’s supposition violates one’s expectations. By familiar reasoning, it therefore follows that

Fitting Quantitative Potential Surprise

x ’s degree d of potential surprise that p fits just in case d is proportional to the degree d' to which p violates x ’s expectations.

For those of us that outright believe that Russell won’t become a bowl of petunias, a significant degree of potential surprise that Russell becomes a bowl of petunias fits. For those of us that merely expect the weather report to be accurate, significant potential surprise that the weather report is inaccurate is an over-reaction and therefore doesn’t fit.

5.3 Potential Relative Surprise

Some philosophers of mind recognise that we can adopt perspectives distinct from our own and that those perspectives can give rise to “relative” emotions—i.e., emotions that are relative to perspectives distinct from our own.²⁵ Given that framework, it is natural to recognise the existence of *potential relative surprise*. Potential relative surprise is like potential surprise, except that it concerns whether our supposition is compatible with expectations distinct from our own (cf. Walton 1990, 259–271). For example, if you get your friend a surprise gift, you can get some enjoyment out of the gift yourself when you feel, in anticipation, their surprise. Here your potential surprise is relative to your friend’s expectations rather than your own. Before we discuss the fittingness conditions of potential relative surprise, it is worthwhile to discuss some further details of the background framework.

²⁵Many authors accept that assumption in some form (e.g., Walton 1990; Gordon 1990, 150; Currie & Ravenscroft 2002; Goldman 2006). These authors differ in whether these states are emotions, quasi-emotions, or offline emotions. These internal debates aren’t important in the present context.

Potential relative surprise arises from a two-step process. First, an agent x adopts expectations B^* distinct from their own. This is explicable in terms of Goldman’s more general notion of e-imagination (e.g., Goldman 2006, 46–48). In particular, an agent x adopts expectations B^* just in case x e-imagines expectation of B^* . This is to engage in a process of mental simulation which outputs a state which resembles expectation of B^* in various respects. The output of this mental simulation is hereafter known as ‘offline expectation’.²⁶ Second, once an agent offline expects B^* , she can “reuse” the surprise process to check whether some supposition p violates her offline expectations B^* . The surprise process then outputs the appropriate degree of potential relative surprise. If it does, then our surprise process outputs potential surprise relative to B^* . Given that proponents of BDTE predict that we can “reuse” BDTE comparators to produce relative emotions (Reisenzein 2009b, 8), potential relative surprise is consonant with the science of surprise.

The difference between potential relative surprise and potential surprise is that the former isn’t relative to one’s own expectations but rather to some distinct perspective B^* . Given that, x ’s potential surprise that p relative to B^* appraises that p violates x ’s offline expectations B^* . It follows that

Fitting Qualitative Potential Relative Surprise

x ’s potential surprise that p relative to B^* fits just in case p violates x ’s offline expectations B^* .

Suppose that you have no expectations about the winning chances of candidates in some political election. Relative to a perspective in which Candidate A has 90% chance to win, most people nonetheless recognise that potential surprise that Candidate A loses fits (Macedo et al. 2004).

Potential relative surprise comes in degrees. The intensity of potential surprise relative to some perspective B^* indicates *to what extent* one’s supposition violates B^* . By familiar reasoning, it follows that

Fitting Quantitative Potential Relative Surprise

x ’s degree d of potential surprise that p relative to B^* fits just in case d is proportional to the degree d' to which p violates x ’s offline expectations B^* .

Suppose that you have no expectations about the distribution of marbles in an urn. Relative to a perspective that includes the proposition that there are 10 red marbles, 40 black marbles, and 50 yellow marbles, it is intuitive that moderate potential surprise that the next chosen marble is red fits, a little potential surprise that the next chosen marble is black fits, and no potential surprise that the next chosen marble is yellow fits. This settles my introduction of the relevant types of surprise and their fittingness conditions.

6 Types of Surprise Test

This section explains to what extent different types of surprise tests implement our solutions to the problem of the premise set and the problem of displacement. §6.1 explains

²⁶The most salient respect in which offline expectation resembles expectation is in its functional connection to our surprise process. Notice that if we suppose a proposition that violates our (online) expectations, then our surprise process normally outputs potential surprise. Similarly, if we suppose a proposition that violates our offline expectation of B^* , then our surprise process normally outputs potential surprise relative to B^* . For further discussion, see Goldman (2006, 149–191).

a basic sort of surprise test that uses potential surprise to evaluate simple circumstantial and epistemic modal statements. §6.2 explains a more sophisticated sort of surprise test that uses potential relative surprise to evaluate complex circumstantial and epistemic modal statements. The distinction between simple and complex modal statements is a technical distinction that receives further discussion below.

6.1 Potential Surprise Test

6.1.1 The Test

Potential Surprise Test (PST) uses states of potential surprise at some proposition p as a guide to p 's modal status.

PST

$(\neg\Diamond)$ Judge p impossible if you have significant potential surprise at p .²⁷

(\Diamond) Judge p possible if you have insignificant potential surprise at p .

To evaluate a modal statement of the form $\text{MODAL}(B)(p)$ with PST, you imagine or suppose its prejacent p . This triggers your surprise process which outputs your degree of potential surprise at the prejacent p . After that, PST uses your potential surprise at the prejacent p as an indication of the modal status of the prejacent p and therefore as a guide to the truth of $\text{MODAL}(B)(p)$ itself.²⁸

PST mentions “significant” and “insignificant” surprise. The thought here is that not just any degree of surprise is sufficient to cause you to judge a proposition impossible. For example, if you expect that John didn’t do it, but you don’t outright believe that, then it’s doubtful that the supposition that John did it is going to give rise to enough surprise to cause you to judge it impossible that John did it. PST is sensitive to this issue. It doesn’t predict that you judge p impossible if supposition of p gives rise to an insignificant degree of surprise. Rather, it requires that supposition of p gives rise to a significant degree of surprise.

This raises an important question. How can we distinguish significant from insignificant degrees of surprise? A natural hypothesis is that whereas violation of an outright belief gives rise to significant surprise, violation of a mere expectation gives rise to insignificant surprise. However, this is too quick. If an agent x over-reacts, then supposition of p can give rise to significant surprise even if p doesn’t violate x 's outright beliefs. Similarly, if an agent x under-reacts, then the supposition of p can give rise to insignificant surprise even if p violates x 's outright beliefs. Nonetheless, this problem has a simple fix. This section argues that *if the relevant type of surprise fits*, then the corresponding surprise tests produce the correct modal judgements. This means that we can restrict ourselves

²⁷These heuristics are formulated in terms of an ‘if’ rather than an ‘if and only if’ because there are multiple different ways to acquire modal knowledge (Williamson 2016b, 800). Note therefore that some might know that Hydrangeas grow in Korea through non-intuitive methods in which surprise plays no role (e.g., through testimony). The examples throughout are chosen to demonstrate the scope and limits of the surprise-based heuristics (rather than out of empirical speculation that these specific examples are known by most people using an intuitive surprise-based heuristic).

²⁸Some have thought that supposition cannot trigger the affect system but that propositional imagination can (Arcangeli 2018, 31–48; but cf. Williamson 2020, 15). But since imagination can also initiate PST, we needn’t resolve that issue here.

to discussion of “normal” cases—i.e., cases in which the relevant type of surprise fits. We therefore operationalize the notions of significant and insignificant surprise as follows.

Significant Surprise

Normally, x has significant potential surprise at p iff p violates x 's outright beliefs.

Insignificant Surprise

Normally, x has insignificant potential surprise at p iff p doesn't violate x 's outright beliefs.

Given facts about outright beliefs, modal epistemologists can use these principles to distinguish significant from insignificant potential surprise.²⁹

6.1.2 Simple Modal Knowledge Acquisition

This section shows that an evaluator can use PST to acquire knowledge of modal statements that are *simple* for her. Some modal statement M is simple for an evaluator x just in case x 's outright beliefs approximate the premise set of M .³⁰ In more intuitive terms, a modal statement is simple if one needn't adopt another perspective to correctly evaluate it.

Consider a concrete example of an epistemic modal statement that is simple for a given evaluator. Notice that if the modal statement is relative to the evaluator's outright beliefs, then that evaluator's outright beliefs approximate the premise set of that modal statement. This can happen, for example, if the evaluator is the speaker of a present tense epistemic modal statement such as (2).

(2) John might've done it.

Suppose that (2) means that SUSPECT (= the proposition that John did it) is possible in view of the speaker's outright beliefs. Given that the speaker just is the evaluator, it follows that SUSPECT is compatible with the speaker's outright beliefs iff SUSPECT is compatible with the evaluator's outright beliefs. The evaluator's outright beliefs therefore approximate the premise set of (2). Now, there are two cases to consider. First, suppose that (2) is true and therefore that SUSPECT is possible in view of the speaker's outright beliefs B . Given Kratzer's Equivalences (see §4), it follows that SUSPECT is compatible with the speaker's outright beliefs B . By Approximation (see §3), it follows that SUSPECT is compatible with the evaluator's outright beliefs B^* . Given that, SUSPECT doesn't violate the evaluator's outright beliefs B^* . If so, supposition of SUSPECT shouldn't give rise to significant potential surprise in the evaluator. Hence, if SUSPECT is possible in view of the speaker's outright beliefs B , then PST correctly recommends the evaluator to judge SUSPECT possible. Second, suppose that (2) is false and therefore

²⁹This framework doesn't attempt to explain suspension of modal judgement or the degrees of confidence we have in our modal judgements. However, if we recognise that some outright beliefs are more entrenched than other outright beliefs (Spohn 2012; Schulz 2021a), then we can recognise that the violation of a more entrenched outright belief gives rise to more significant surprise than the violation of a less entrenched outright belief. Plausibly, if supposition of p gives rise to the lowest degree of significant surprise, then the evaluator has low confidence that p is impossible and perhaps suspends judgement. These issues are left to another occasion.

³⁰Strictly speaking, a modal statement M in a context c is simple for an evaluator x just in case x 's outright beliefs approximate the premise set of M in c . As I said in §2, the premise set of a modal statement is a context-dependent matter. We can set this aside for the moment.

that SUSPECT is impossible in view of the speaker's outright beliefs B . Given Kratzer's Equivalences, it follows that SUSPECT is incompatible with the speaker's outright beliefs B . By Approximation, it follows that SUSPECT is incompatible with the evaluator's outright beliefs B^* . Given that, SUSPECT violates the evaluator's outright beliefs B^* . If so, supposition of SUSPECT should give rise to significant potential surprise in the evaluator. Hence, if SUSPECT is impossible in view of the speaker's outright beliefs B , then PST correctly recommends the evaluator to judge SUSPECT impossible.

Consider a concrete example of a circumstantial modal statement that is simple for a given evaluator Eve.

- (1) Hydrangeas can grow in Korea. (Kratzer 1977)

Suppose that (1) means that HYDRANGEAS (= that at some future time, hydrangeas grow in Korea) is possible in view of the relevant botanical and meteorological circumstances. Notice that if Eve has reasonably accurate outright beliefs about the relevant circumstances, then Eve's outright beliefs can approximate the premise set of (1). If so, then HYDRANGEAS is compatible with Eve's outright beliefs *iff* HYDRANGEAS is compatible with the relevant circumstances B . Now, there are two cases to consider. First, suppose that (1) is true and therefore that HYDRANGEAS is possible in view of the relevant circumstances B . Given Kratzer's Equivalences, it follows that HYDRANGEAS is compatible with the relevant circumstances B . By Approximation, it follows that HYDRANGEAS is compatible with Eve's outright beliefs B^* . Given that, HYDRANGEAS doesn't violate Eve's outright beliefs B^* . If so, supposition of HYDRANGEAS shouldn't give rise to significant potential surprise in Eve. Hence, if HYDRANGEAS is possible in view of the relevant circumstances B , then PST correctly recommends Eve to judge HYDRANGEAS possible. Second, suppose that (1) is false and therefore that HYDRANGEAS is impossible in view of the relevant circumstances B . Given Kratzer's Equivalences, it follows that HYDRANGEAS is incompatible with the relevant circumstances B . By Approximation, it follows that HYDRANGEAS is incompatible with Eve's outright beliefs B^* . Given that, HYDRANGEAS violates Eve's outright beliefs B^* . If so, supposition of HYDRANGEAS should give rise to significant potential surprise in Eve. Hence, if HYDRANGEAS is impossible in view of the relevant circumstances B , then PST correctly recommends Eve to judge HYDRANGEAS impossible.³¹

6.1.3 The Limits

PST isn't reliable no matter what. It has limits. PST is not a reliable guide to the truth-values of *complex* modal statements. Some modal statement M is complex for an evaluator x just in case x 's outright beliefs don't approximate the premise set of M .³² In

³¹Note that (1) is simple even for some evaluators that outright believe that at present there are no hydrangeas in Korea. If an evaluator x outright believes that, then x 's supposition of the proposition that *at some future time, hydrangeas grow in Korea* won't necessarily give rise to significant potential surprise. This is because the proposition that *at present no hydrangeas grow in Korea* is compatible with the proposition that *at some future time, hydrangeas grow in Korea*. However, if an evaluator y outright believes that *there is no future time at which hydrangeas grow in Korea*, then (1) is not simple for y . If so, PST is not a reliable guide to the truth of (1) for y . I am going to discuss this limitation of PST in §6.1.3.

³²Again, strictly speaking, a modal statement M in a context c is complex for an evaluator x just in case x 's outright beliefs don't approximate the premise set of M in c . We can set this aside for the moment.

more intuitive terms, a modal statement is complex if one has to adopt another perspective to correctly evaluate it.³³ A modal statement that is simple for one evaluator is often complex for another evaluator because outright beliefs often differ between evaluators.

If a modal statement M is complex for an evaluator, then PST can give incorrect results. For example, suppose that (2) is true and therefore that SUSPECT is possible in view of the detective's outright beliefs. Suppose further that John himself is the hearer of (2) and he outright believes that he didn't commit the murder. If so, (2) is complex for John because SUSPECT is incompatible with John's outright beliefs. Given that, SUSPECT violates John's outright beliefs. If so, supposition of SUSPECT should give rise to significant potential surprise in John. Hence, if (2) is complex for John, then PST recommends John to judge SUSPECT impossible. But (2) is true. John might've done it in view of the detective's beliefs.

Further, suppose that (1) is true and therefore that HYDRANGEAS is possible in view of the relevant circumstances. Suppose further that some evaluator, Eliot, outright believes that Korea will always be a barren desert, that hydrangeas will never grow in a barren desert, and therefore that there is no future time at which hydrangeas grow in Korea. If so, (1) is complex for Eliot because HYDRANGEAS is incompatible with Eliot's outright beliefs. By familiar reasoning, PST recommends Eliot to judge HYDRANGEAS impossible. But (1) is true. Hydrangeas can grow in Korea in view of the relevant circumstances.

Further examples of complex modal statements arise from the interaction between modals and tense.³⁴ Due to modal/tense interactions, some epistemic modal statements are complex for speakers themselves. For example, imagine that Sophie is looking for some ice cream and checks the freezer (von Fintel & Gillies 2008). She sees that there is no ice cream left. But now, suppose that Sophie tries to explain her decision to check the freezer by uttering (3).

(3) There might have been ice cream in the freezer.

(3) says that FREEZER (= the proposition that there is ice cream in the freezer) is possible in view of Sophie's *past* beliefs. But Sophie (at the utterance time) knows and therefore outright believes that there is no ice cream in the freezer. If so, (3) is complex for Sophie because FREEZER is incompatible with her current outright beliefs. By familiar reasoning, PST recommends her to judge FREEZER impossible. But (3) is true. There might have been ice cream in the freezer in view of Sophie's past beliefs.

Moreover, tense/modal interactions can give rise to circumstantial modal statements that are complex even for evaluators that are knowledgeable about the present circumstances. For example, imagine that Donald knows that Barack Obama became President of the USA in 2009. Donald considers this regrettable and therefore wistfully utters (4).

(4) It could have been the case that Barack Obama never became President.

(cf. Dummett 1973, 131)

(4) draws our attention to some circumstances at some past time t before Obama became President (e.g., $t = 2008$). Its prejacent is NEVER-PRESIDENT (= the proposition that there is no future time t_{+n} at which Obama becomes President). (4) says that NEVER-

³³For several examples of complex modal statements, see Bach (2011, 40–46).

³⁴For discussion of the interaction between modals and tense, see Rullmann & Matthewson (2018).

PRESIDENT is possible in view of those past circumstances.³⁵ But Donald knows and therefore outright believes that there is a time t_{+n} at which Obama becomes President (e.g., $t_{+n} = 2009$). If so, (4) is complex for Donald because NEVER-PRESIDENT is incompatible with his outright beliefs. By familiar reasoning, PST recommends him to judge NEVER-PRESIDENT impossible. But (4) is possible. It could have been the case that Barack Obama never became President in view of the relevant past circumstances.

Here is another type of limitation of PST. PST isn't reliable regardless of whether one's potential surprise fits. PST can produce errors of under-generation. Suppose that p is in fact possible. If your surprise process misfires and produces unfitting significant potential surprise at p , then PST misguides you to judge p impossible. Further, PST can produce errors of over-generation. Suppose that p is in fact impossible. If your surprise process misfires and produces unfitting insignificant potential surprise at p , then PST misguides you to judge p possible. However, given that reliable detection of violation of expectation is crucial to the maintenance of an adaptive cognitive system (Reisenzein et al. 2019), cases of potential surprise that don't fit are rare.

We have just seen that PST isn't reliable if the evaluator's outright beliefs don't approximate the premise set. Nor is PST reliable if one's potential surprise doesn't fit. But that isn't grounds to denigrate PST. After all, deduction isn't reliable regardless of whether the premises are true (Goldman 1979, 98). But deduction is a method of knowledge acquisition *par excellence*. Still, we have to recognise that PST has its limitations. PST cannot underlie our knowledge of complex modal statements. However, there is a potential *relative* surprise test that is of distinctive usefulness in just these sorts of cases.³⁶

6.2 Potential Relative Surprise Test

6.2.1 The Test

Potential Relative Surprise Test (PRST) uses states of potential surprise at some proposition p relative to some perspective B^* as a guide to p 's modal status in view of some premise set B .

³⁵(4) is a past tense circumstantial modal statement (cf. 'could have been'), which implies that its premise set concerns the circumstances at some past time (e.g., Rullmann & Matthewson 2018, 282, 300). Further, (4) is a circumstantial modal statement, which implies that its prejacent concerns a time which is future with respect to the relevant circumstances (e.g., Rullmann & Matthewson 2018, 300–303).

³⁶A reviewer suggests that my account doesn't give a determinate verdict about whether an agent that believes an impossible proposition $\neg p$, supposes p , and undergoes potential surprise that p ought to conclude from their potential surprise either (i) that p is impossible or (ii) that the content of their belief that $\neg p$ is impossible. In response, assume that the impossible proposition $\neg p$ is not part of the relevant premise set, or an approximation of that premise set (cf. Kratzer 1977, 347–355). If so, my account predicts that the agent cannot reliably use PST to evaluate the relevant modal claim. This is because the agent's expectations don't approximate the premise set. Put otherwise, this modal claim is complex for the agent that the reviewer describes. As I've argued, PST is unreliable with respect to evaluation of complex modal statements.

PRST

- ($\neg\Diamond$) Judge some proposition p impossible in view of B if you have significant potential surprise at p relative to B^* .
- (\Diamond) Judge some proposition p possible in view of B if you have insignificant potential surprise at p relative to B^* .

To evaluate a modal statement $\text{MODAL}(B)(p)$ with PRST, you first adopt a perspective B^* that approximates B . After that, you imagine or suppose its prejacent p . This triggers your surprise process which outputs your degree of potential surprise at the prejacent p relative to B^* . After that, PRST uses your potential surprise at p relative to B^* as an indication of the modal status of the prejacent p in view of B and therefore as a guide to the truth of $\text{MODAL}(B)(p)$ itself.

PRST requires the evaluator to adopt a perspective B^* that approximates B . This raises an important question. If M is complex for x , then how does x adopt one of the various perspectives that approximate the premise set of M ? It is important to appreciate that evaluators that are in a position to know a modal statement M are not completely in the dark about its premise set and its departures from their own perspective (cf. Williamson 2016b, 797–798). For example, Sophie’s use of ‘might’ in (3) is some indication that the relevant premise set consists of her information (rather than the circumstances).

- (3) There might have been ice cream in the freezer.

Further, Sophie’s use of ‘might have been’ is some indication that the relevant premise set consists of her information at a relevant past time. More importantly, evaluators often have access to various contextual clues about the premise sets of the relevant modal statement.³⁷ Suppose that Sophie utters (3) as the answer to the question ‘why were you looking in the freezer?’ The hearer can use that information to determine that the premise set of (3) consists of Sophie’s information at the time of her decision to search the freezer.³⁸ An evaluator that is sensitive to these sorts of issues is *competent* with the premise set of the relevant modal statement. It is plausible that a competent evaluator is able to make minimal adjustments to her own perspective—guided by said competence—in order to adopt a perspective that approximates the premise set (e.g., Nickerson 1999; Goldman 2006, 149–150). Given this “anchor and adjustment” model of perspective shift, I am now going to show that an agent can use PRST to acquire knowledge of modal statements that are complex for her.

6.2.2 Complex Modal Knowledge Acquisition

We begin with a complex epistemic modal statement (3), which means that the prejacent proposition FREEZER is possible in view of Sophie’s outright beliefs at the relevant past time B . To ensure that (3) is complex, suppose that the evaluator knows and therefore outright believes that there is no ice cream in the freezer. Suppose further that (3) is true

³⁷This is true, for example, if the conversational context is non-defective. A non-defective context “is a context in which the participants’ beliefs about the common ground are all correct” (Stalnaker 2002, 717).

³⁸We can make similar points with respect to (4). Donald’s use of ‘could’ indicates that the circumstances are at issue, his use of ‘could have been’ indicates that past circumstances are at issue, and contextual clues indicate that the relevant time precedes Obama’s inauguration.

and therefore that FREEZER is possible in view of Sophie’s outright beliefs at the relevant past time B . Now, to know (3) through PRST, the evaluator must first adopt some perspective B^* that approximates Sophie’s past perspective B . Guided by his knowledge of the context, the evaluator must therefore abstract from his own present outright belief that there isn’t ice cream in the freezer. For the purposes of the evaluation of (3), the rest of the evaluator’s perspective can remain in place regardless of whether it conflicts in irrelevant ways with Sophie’s past perspective (Nickerson 1999). If the evaluator manages that moderate feat of the imagination, then his expectations approximate the premise set of (3). By familiar reasoning, PRST therefore correctly recommends the evaluator to judge FREEZER possible.³⁹

Turn now to a complex circumstantial modal statement (4), which means that the pre-jacent proposition NEVER-PRESIDENT is possible in view of circumstances at the relevant past time.

- (4) It could have been the case that Barack Obama never became President.

To ensure that (4) is complex, suppose that the evaluator knows and therefore outright believes that Obama became President in 2009. Suppose further that (4) is true and therefore that NEVER-PRESIDENT is possible in view of the relevant past circumstances. Now, to know (4) through PRST, the evaluator must first adopt some perspective B^* that approximates the relevant past circumstances B . Guided by his knowledge of the context, the evaluator must therefore abstract from his own present outright belief that Obama became President in 2009. For the purposes of the evaluation of (4), the rest of the evaluator’s perspective can remain in place regardless of whether it conflicts in irrelevant ways with the relevant past circumstances. If the evaluator manages that moderate feat of the imagination, then his expectations approximate the premise set of (4). By familiar reasoning, PRST therefore correctly recommends the evaluator to judge NEVER-PRESIDENT possible.

6.2.3 The Limits

PRST isn’t reliable no matter what. Reliable use of PRST often requires one to adopt a perspective distinct from one’s own. But as we’ve seen above, when we take another perspective, we assume that our own perspective is a reasonable initial model of other perspectives and adapt it in minimal ways to capture the relevant departures (cf. Stalnaker 1968; Lewis 1973). But as psychologists and philosophers have shown, over-reliance on one’s own perspective can cause various egocentric biases (e.g., Davis et al. 1986; Gordon 1990; Nickerson 1999; Epley et al. 2004; Goldman 2006; Clavel Vázquez & Clavel-Vázquez 2023; Jones & Schoonen 2026). Hence, if the surprise account is correct, we ought to expect that these egocentric biases can cause various modal biases. The point for present purposes is that PRST inherits the limitations of our capacities to adopt distinct perspectives. But since we aren’t omniscient with respect to modal matters, that isn’t reason to

³⁹Note that if (3) is false and therefore that FREEZER is impossible in view of Sophie’s outright beliefs at the relevant past time B , then (3) is simple for the given evaluator. Given that the evaluator’s outright beliefs are incompatible with FREEZER, he needn’t adjust his perspective. By familiar reasoning, PRST correctly recommends him to judge FREEZER impossible. As I flagged above, what is simple/complex for an evaluator is, strictly speaking, a modal statement *in a context* c . The same point applies to my discussion of (4) below.

doubt the surprise account. The empirical question for future research is to what extent modal cognition shares the signature limits of PRST.⁴⁰

7 Conclusion

This paper began with two problems that modal semantics suggests that creatures like us face in our acquisition of modal knowledge. The problem of the premise set requires knowers to establish epistemic access to the premise set of modal statements we know. The problem of displacement requires knowers to establish epistemic access to accessible but non-actual possible worlds. The rest of the paper argued that various types of surprise are at the heart of our solution to those problems. To evaluate a modal statement, we accept an approximation of its premise set, we suppose its prejacent, and check whether that gives rise to significant potential surprise relative to that premise set. Given that the surprise account can implement our solutions to the cognitive problems that modal knowledge acquisition presents, and can capture the intuitive nature of modal knowledge acquisition, it outlines a possible and empirically plausible way of acquiring modal knowledge. As Williamson (2021, 192) once put it, the “default hypothesis is that what we have identified as capable of doing the job is what does do the job”.^{41,42}

The consequences of the surprise account of modal knowledge acquisition are significant. Emotions, it turns out, are at the heart of modal knowledge acquisition. This vindicates epistemologies that have given the sentiments a more central role in our cognitive lives (Hume 2007; Smith 1976). In addition, given the important role of modal knowledge in some of our most central philosophical methods (e.g., the method of cases), it suggests that philosophical practice itself is, in part, emotional. Moreover, the surprise

⁴⁰For example, younger children are subject to egocentric biases that privilege their own perspective regardless of whether that is appropriate (e.g., Robinson et al. 2006). The surprise account therefore predicts that younger children are going to struggle to evaluate complex modal statements. In fact, several developmental studies suggest that children struggle to evaluate the modal status of propositions that violate their core expectations (e.g., Shtulman & Carey 2007; Shtulman 2009; Lane et al. 2016). More in-depth engagement with the psychological literature is left to another occasion.

⁴¹Note that my conclusion doesn’t suggest that we don’t acquire modal knowledge in other ways, nor does it ignore the role of supposition and imaginative shift of one’s perspective in modal knowledge acquisition as these have a role alongside surprise in the surprise heuristics themselves. As I’ve highlighted in §6.2.3, one’s capacities to adopt perspectives distinct from one’s own might limit one’s modal knowledge acquisition. Maybe that is sometimes a feature rather than a limitation. Maybe we cannot or do not adopt perspectives that involve metaphysical or logical impossibilities (e.g., perspectives that include the proposition that there is a round square).

⁴²How similar is the surprise account to previous accounts of modal knowledge acquisition? I am not able to answer this question in full here, but here are some suggestive remarks. The surprise account is closest to “OBR” accounts of modal knowledge acquisition that involve offline belief revision (e.g., Berto 2021; Schoonen 2024). Both accounts involve mental simulation, imaginative shift of one’s own perspective, and cognitive systems that are part of, or enable, the agent’s belief revision system. However, whereas the surprise account involves simulation of a peripheral system that checks whether the agent ought to initiate belief revision, OBR accounts involve simulation of the belief revision system itself. Relatedly, the surprise account is static. Unlike OBR accounts, it doesn’t require the agent to “develop a supposition”—i.e., to transition between imaginative stages in a wider imaginative episode (cf. Langland-Hassan 2016). Further, the notion of supposition in these two accounts is different. Whereas supposition is offline learning in the surprise account, supposition is offline updating in OBR accounts (cf. fn.24 above). Further, unlike OBR accounts, the surprise account doesn’t involve objectual imagination—i.e., it doesn’t require the agent to imagine a situation that is a truth-maker of the relevant modal statement (cf. Yablo 1993, 27; Chalmers 2002, 150; Berto 2021, S2031). As a final note, it isn’t obvious to me that these theories are in competition, as we can acquire modal knowledge in different ways.

account supports a kind of anti-exceptionalism (Williamson 2007). Any creature that is capable of surprise is capable of simple modal cognition through the use of potential surprise test. The account might therefore have applications to the modal cognition of infants (Goddu et al. 2021), younger children (Shtulman & Carey 2007), and non-human animals (Engelmann et al. 2023). Finally, the surprise account is a contribution to a new “naturalistic” methodological project that is emerging in the epistemology of modality (e.g. Boardman & Schoonen 2023, 2025; Jones & Schoonen 2026). This project brings together relevant results from across the cognitive sciences (especially linguistics and psychology) and uses those results to inform both its conception of the target (i.e., the truth-conditions of modal statements) and of the means we use to hit that target (e.g., surprise).

References

- Arcangeli, M. (2018). *Supposition and the Imaginative Realm: A Philosophical Inquiry*. Routledge.
- Bach, K. (2011). Perspectives on possibilities: Contextualism, relativism, or what? In Egan, A. & Weatherson, B. (Eds.), *Epistemic Modality*, (pp. 19–59). Oxford University Press.
- Basu, R. (2023). The Ethics of Expectations. In Timmons, M. (Ed.), *Oxford Studies in Normative Ethics*, vol. 13, (pp. 149–169). Oxford University Press.
- Benton, M. A. & Turri, J. (2014). Iffy predictions and proper expectations. *Synthese*, **191**, 1857–1866.
- Berto, F. (2021). Taming the runabout imagination ticket. *Synthese*, **198**(8), 2029–2043. doi:10.1007/s11229-018-1751-6.
- Blackburn, S. (1993). Morals and Modals. In *Essays in Quasi-realism*, (pp. 52–74). Oxford University Press.
- Boardman, S. & Schoonen, T. (2023). Epistemologists of Modality Wanted. *Synthese*, **202**(6), 192. doi:10.1007/s11229-023-04416-8.
- (2025). Core Imagination. *Philosophers’ Imprint*, **25:29**.
- Bradley, R. (2017). *Decision Theory with a Human Face*. Cambridge University Press.
- Carey, S. (2009). *The Origin of Concepts*. Oxford University Press.
- Cariani, F. (2021). *The Modal Future: A Theory of Future-directed Thought and Talk*. Cambridge University Press.
- Chalmers, D. (2002). Does Conceivability Entail Possibility? In Gendler, T. S. & Hawthorne, J. (Eds.), *Conceivability and Possibility*, (pp. 145–200). Oxford University Press.
- Clavel Vázquez, M. J. & Clavel-Vázquez, A. (2023). Robustly embodied imagination and the limits of perspective-taking. *Philosophical Studies*, **180**(4), 1395–1420. doi:10.1007/s11098-023-01936-4.
- Currie, G. & Ravenscroft, I. (2002). *Recreative Minds*. Oxford University Press.
- D’Arms, J. & Jacobson, D. (2000). The Moralistic Fallacy: On the “Appropriateness” of Emotions. *Philosophy and Phenomenological Research*, **61**(1), 65–90.
- Darwin, C. (1872). *The Expression of the Emotions in Man and Animals*. John Murray.
- Davis, H. L., Hoch, S. J., & Ragsdale, E. K. E. (1986). An Anchoring and Adjustment Model of Spousal Predictions. *Journal of Consumer Research*, **13**(1), 25–37.

- Dummett, M. (1973). *Frege: Philosophy of Language*. Harper & Row.
- Ekman, P. (1992). An argument for basic emotions. *Cognition & Emotion*, **6**(3-4), 169–200.
- Engelmann, J. M., Völter, C. J., Goddu, M. K., Call, J., Rakoczy, H., & Herrmann, E. (2023). Chimpanzees prepare for alternative possible outcomes. *Biology Letters*, **19**(6), 20230179. doi:10.1098/rsbl.2023.0179.
- Epley, N., Keysar, B., Van Boven, L., & Gilovich, T. (2004). Perspective taking as egocentric anchoring and adjustment. *Journal of Personality and Social Psychology*, **87**(3), 327–339. doi:10.1037/0022-3514.87.3.327.
- Eva, B., Shear, T., & Fitelson, B. (2022). Four approaches to supposition. *Ergo*, **8**(26).
- Fischer, B. & Leon, F. (2017). *Modal Epistemology After Rationalism*. Springer.
- Gendler, T. S. & Hawthorne, J. (2002). *Conceivability and Possibility*. Oxford University Press.
- Gigerenzer, G. E., Hertwig, R. E., & Pachur, T. E. (Eds.) (2011). *Heuristics: The Foundations of Adaptive Behavior*. Oxford University Press.
- Goddu, M. K., Sullivan, J. N., & Walker, C. M. (2021). Toddlers learn and flexibly apply multiple possibilities. *Child Development*, **92**(6), 2244–2251. doi:10.1111/cdev.13668.
- Goldman, A. I. (1979). What is Justified Belief. In Pappas, G. (Ed.), *Justification and Knowledge*, (pp. 1–25). D. Reidel.
- (2006). *Simulating Minds: The Philosophy, Psychology, and Neuroscience of Mindreading*. Oxford University Press.
- Gordon, R. M. (1990). *The Structure of Emotions: Investigations in Cognitive Philosophy*. Cambridge University Press.
- Haidt, J. (2001). The Emotional Dog and its Rational Tail: A Social Intuitionist Approach to Moral Judgment. *Psychological Review*, **108**(4), 814–834. doi:10.1037/0033-295x.108.4.814.
- Howard, C. (2018). Fittingness. *Philosophy Compass*, **13**(11), e12542.
- Hume, D. (2007). *A Treatise of Human Nature: A Critical Edition*. Oxford University Press.
- Ichikawa, J. (2016). Modals and Modal Epistemology. In Kind, A. & Kung, P. (Eds.), *Knowledge Through Imagination*, (pp. 124–144). Oxford University Press.
- Jenkins, C. S. (2008). Modal Knowledge, Counterfactual Knowledge and the Role of Experience. *The Philosophical Quarterly*, **58**(233), 693–701. doi:10.1111/j.1467-9213.2008.579.x.
- Jones, M. & Schoonen, T. (2026). Embodied simulation and knowledge of possibilities. *Philosophical Psychology*, **39**(2), 522–546. doi:10.1080/09515089.2024.2417990.
- Joyce, J. M. (1999). *The Foundations of Causal Decision Theory*. Cambridge University Press.
- Judge, J. (2018). The Surprising Thing About Musical Surprise. *Analysis*, **78**(2), 225–234. doi:10.1093/analys/anx139.
- Kahneman, D., Slovic, P., & Tversky, A. (1982). *Judgment Under Uncertainty: Heuristics and Biases*. Cambridge University Press.
- Kind, A. (2021). Can imagination be unconscious? *Synthese*, **199**(5-6), 13121–13141.
- Knobe, J. & Szabó, Z. G. (2013). Modals with a Taste of the Deontic. *Semantics and Pragmatics*, **6**(1), 1–42.

- Kratzer, A. (1977). What ‘Must’ and ‘Can’ Must and Can Mean. *Linguistics and Philosophy*, **1**(3), 337–355.
- (2012). *Modals and Conditionals: New and Revised Perspectives*. Oxford University Press.
- Lane, J. D., Ronfard, S., Francioli, S., & Harris, P. L. (2016). Children’s imagination and belief: Prone to flights of fancy or grounded in reality? *Cognition*, **152**, 127–140.
- Langland-Hassan, P. (2016). On choosing what to imagine. In Kind, A. & Kung, P. (Eds.), *Knowledge through Imagination*. Oxford University Press.
- Levi, I. (1967). *Gambling with Truth: An Essay on Induction and the Aims of Science*. Knopf.
- (2004). *Mild Contraction: Evaluating Loss of Information Due to Loss of Belief*. Oxford University Press.
- Lewis, D. (1973). *Counterfactuals*. Blackwell.
- (1979). Attitudes De Dicto and De Se. *The Philosophical Review*, **88**(4), 513–543.
- (1984). Letter from David Lewis to Colin McGinn. David Lewis Papers, C1520, ‘McGinn, Colin’, Box B-000670 Folder 13, Princeton University Library.
- Macedo, L. & Cardoso, A. (2019). A contrast-based computational model of surprise and its applications. *Topics in cognitive science*, **11**(1), 88–102.
- Macedo, L., Reisezein, R., & Cardoso, A. (2004). Modeling forms of surprise in artificial agents: Empirical and theoretical study of surprise functions. In Forbus, K., Gentner, D., & Regier, T. (Eds.), *Proceedings of the Annual Meeting of the 26th Annual Conference of the Cognitive Science Society*, vol. 26, (pp. 873–878). Erlbaum.
- McGinn, C. (1975). Modal Reality. In Healey, R. (Ed.), *Reduction, Time and Reality*, (pp. 65–108). Cambridge University Press.
- Miceli, M. & Castelfranchi, C. (2015). *Expectancy and Emotion*. Oxford University Press.
- Millican, P. (2020). Hume on Modality. In *The Routledge Handbook of Modality*, (pp. 364–377). Routledge.
- Mondadori, F. & Morton, A. (1976). Modal Realism: The Poisoned Pawn. *Philosophical Review*, **85**(1), 3–20.
- Munnich, E. L., Foster, M. I., & Keane, M. T. (2019). Editors’ introduction and review: An appraisal of surprise: Tracing the threads that stitch it together. *Topics in Cognitive Science*, **11**(1), 37–49.
- Munnich, E. L. & Ranney, M. A. (2019). Learning from surprise: Harnessing a metacognitive surprise signal to build and adapt belief networks. *Topics in cognitive science*, **11**(1), 164–177.
- Nickerson, R. S. (1999). How we know—and sometimes misjudge—what others know: Imputing one’s own knowledge to others. *Psychological bulletin*, **125**(6), 737.
- Peacocke, C. (1999). *Being Known*. Clarendon Press.
- Pezzo, M. (2003). Surprise, defence, or making sense: What removes hindsight bias? *Memory*, **11**(4-5), 421–441.
- Phillips, J. & Knobe, J. (2018). The Psychological Representation of Modality. *Mind and Language*, **33**(1), 65–94. doi:10.1111/mila.12165.
- Portner, P. (2009). *Modality*. Oxford University Press.

- Reisenzein, R. (2000). The subjective experience of surprise. In Bless, H. & Forgas, J. P. (Eds.), *The Message Within: The Role of Subjective Experience in Social Cognition and Behavior*, (pp. 262–279). Psychology Press.
- (2009a). Emotional experience in the computational belief–desire theory of emotion. *Emotion Review*, **1**(3), 214–222.
- (2009b). Emotions as metarepresentational states of mind: Naturalizing the belief–desire theory of emotion. *Cognitive Systems Research*, **10**(1), 6–20.
- Reisenzein, R., Horstmann, G., & Schützwohl, A. (2019). The cognitive-evolutionary model of surprise: A review of the evidence. *Topics in Cognitive Science*, **11**(1), 50–74.
- Reisenzein, R., Meyer, W.-U., & Niepel, M. (2012). Surprise. In Ramachandran, V. S. (Ed.), *Encyclopedia of Human Behavior*. Academic Press, second ed.
- Robinson, E. J., Rowley, M. G., Beck, S. R., Carroll, D. J., & Apperly, I. A. (2006). Children’s Sensitivity to Their Own Relative Ignorance: Handling of Possibilities Under Epistemic and Physical Uncertainty. *Child Development*, **77**(6), 1642–1655.
- Roca-Royes, S. (2017). Similarity and possibility: An epistemology of de re possibility for concrete entities. In Fischer, B. & Leon, F. (Eds.), *Modal Epistemology After Rationalism*, vol. 378, (pp. 221–245). Springer.
- (2021). The Integration Challenge. In Bueno, O. & Shalkowski, S. (Eds.), *The Routledge Handbook of Modality*, (pp. 157–166). Routledge.
- Rosen, G. (2015). The Alethic Conception of Moral Responsibility. In Clarke, R., McKenna, M., & Smith, A. M. (Eds.), *The Nature of Moral Responsibility: New Essays*, (pp. 65–88). Oxford University Press.
- Rullmann, H. & Matthewson, L. (2018). Towards a theory of modal-temporal interaction. *Language*, **94**(2), 281–331.
- Schoonen, T. (2024). A Note on the Epistemological Value of Pretense Imagination. *Episteme*, **21**(1), 99–118. doi:10.1017/epi.2021.2.
- Schulz, M. (2021a). Practical reasoning and degrees of outright belief. *Synthese*, **199**(3-4), 8069–8090.
- (2021b). Strong knowledge, weak belief? *Synthese*, **199**(3-4), 8741–8753.
- Searle, J. (1983). *Intentionality: An Essay in the Philosophy of Mind*. Oxford University Press.
- Shackle, G. L. S. (1949). *Expectation in Economics*. Cambridge University Press.
- (1961). *Decision, Order and Time in Human Affairs*. Cambridge University Press.
- Shtulman, A. (2009). The development of possibility judgment within and across domains. *Cognitive Development*, **24**(3), 293–309.
- Shtulman, A. & Carey, S. (2007). Improbable or Impossible? How Children Reason About the Possibility of Extraordinary Events. *Child Development*, **78**(3), 1015–1032.
- Smith, A. (1976). The Theory of Moral Sentiments. In Raphael, D. D. & Macfie, A. L. (Eds.), *The Glasgow Edition of the Works and Correspondence of Adam Smith*, vol. 1. Oxford University Press.
- Spelke, E. S. (1990). Principles of object perception. *Cognitive science*, **14**(1), 29–56.
- Spohn, W. (2006). Isaac Levi’s Potentially Surprising Epistemological Picture. In Olsson, E. J. (Ed.), *Knowledge and Inquiry: Essays on the Pragmatism of Isaac Levi*, (pp. 125–142). Cambridge University Press.

- (2012). *The Laws of Belief: Ranking Theory and Its Philosophical Applications*. Oxford University Press.
- Stalnaker, R. (1968). A Theory of Conditionals. In Rescher, N. (Ed.), *Studies in Logical Theory (American Philosophical Quarterly Monographs 2)*, (pp. 98–112). Blackwell.
- (1984). *Inquiry*. Cambridge University Press.
- (2002). Common Ground. *Linguistics and Philosophy*, **25**(5-6), 701–721.
- Strohming, M. & Yli-Vakkuri, J. (2019). Knowledge of objective modality. *Philosophical Studies*, **176**(5), 1155–1175. doi:10.1007/s11098-018-1052-4.
- Tappolet, C. (2011). Values and Emotions: Neo-Sentimentalism’s Prospects. In Bagnoli, C. (Ed.), *Morality and the Emotions*, (pp. 117–134). Oxford University Press.
- Teigen, K. H. & Keren, G. (2003). Surprises: Low probabilities or high contrasts? *Cognition*, **87**(2), 55–71.
- Vetter, B. (2016). Williamsonian modal epistemology, possibility-based. *Canadian Journal of Philosophy*, **46**(4-5), 766–795.
- von Fintel, K. & Gillies, A. S. (2008). CIA leaks. *Philosophical Review*, **117**(1), 77–98.
- (2011). ‘Might’ Made Right. In Egan, A. & Weatherson, B. (Eds.), *Epistemic Modality*, (pp. 108–130). Oxford University Press.
- von Fintel, K. & Heim, I. (2011). Intensional Semantics.
- Walton, K. L. (1990). *Mimesis as Make-believe: On the Foundations of the Representational Arts*. Harvard University Press.
- Williamson, T. (2000). *Knowledge and Its Limits*. Oxford University Press.
- (2007). *The Philosophy of Philosophy*. Wiley-Blackwell.
- (2016a). Modal science. *Canadian Journal of Philosophy*, **46**(4-5), 453–492.
- (2016b). Reply to Vetter. *Canadian Journal of Philosophy*, **46**(4-5), 796–802.
- (2020). *Suppose and Tell: The Semantics and Heuristics of Conditionals*. Oxford University Press.
- (2021). The Counterfactual-Based Approach to Modal Epistemology. In Bueno, O. & Shalkowski, S. (Eds.), *The Routledge Handbook of Modality*, (pp. 188–197). Routledge.
- Yablo, S. (1993). Is Conceivability a Guide to Possibility? *Philosophy and Phenomenological Research*, **53**(1), 1–42. doi:10.2307/2108052.
- Yalcin, S. (2011). Nonfactualism About Epistemic Modality. In Egan, A. & Weatherson, B. (Eds.), *Epistemic Modality*. Oxford University Press.
- (2018). Belief as question-sensitive. *Philosophy and Phenomenological Research*, **97**(1), 23–47.